Inferring Rankings Under Constrained Sensing

Srikanth Jagabathula   Devavrat Shah *

Abstract

We consider the problem of recovering a function on the space of permutations of n elements, denoted by S_n, using *various forms* of partial information. This problem naturally arises in various applications such as elections, ranking teams in a sports league, recommendation systems, etc. To begin with, we classify *various forms* of partial information through it's natural relation to various types of partitions of integer n. This is inspired by the representations of group S_n. As the main result, we provide a tight or near tight characterization of the conditions under which it is possible to recover a function using each form of partial information; the characterization is in terms of the sparsity (cardinality of the support of the function) that can be recovered. Under sufficiency conditions for recovery, we propose a novel and iterative algorithm to recover the function for each form of partial information. Moreover, our algorithm implicitly solves an appropriate ℓ_0 minimization problem, establishing that the function to be recovered has the sparsest support that is consistent with given data. Under necessity conditions for recovery, we utilize an information theoretic Fano's inequality to establish bounds on sparsity that cannot be recovered. In the process, we obtain a characterization of the number of ways in which a doubly-stochastic matrix can be decomposed into permutation matrices.

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*SJ and DS are with Laboratory for Information and Decision Systems, Department of EECS, Massachusetts Institute of Technology. Emails: `jskanth,devavrat@mit.edu`